Shop Scheduling

Applications

- Model Factory-like Settings
- Also models packet routing
- ...

Basic Model: Multiple machines. A jobs consist of operations, each operations has a

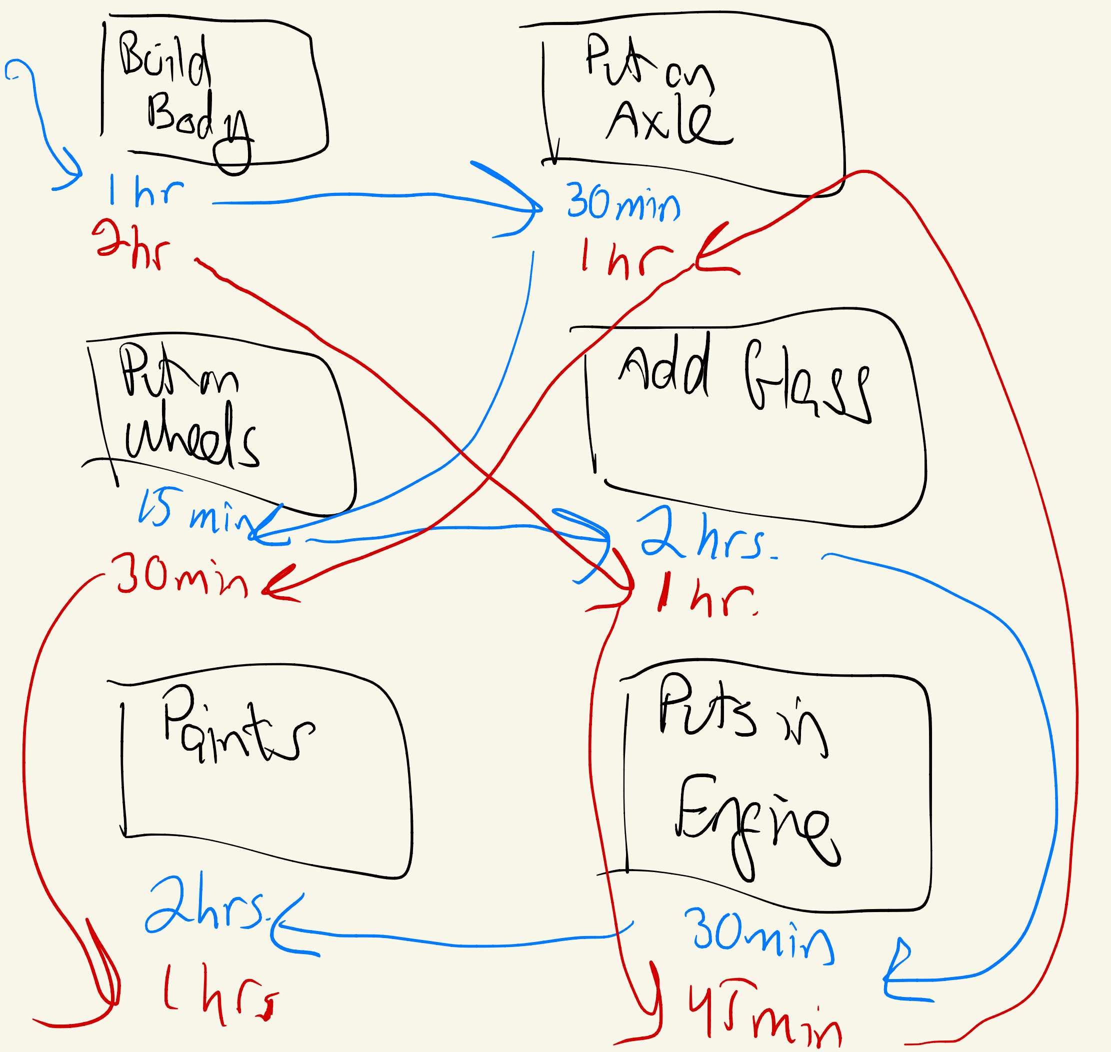
- processing time p_{ij}
- Machine on which to run M_{ij}

Car Factory

Job1 = make a car 1

Job2 = make car 2

⋮



Shop Scheduling

Applications

- Model Factory-like Settings
- Also models packet routing
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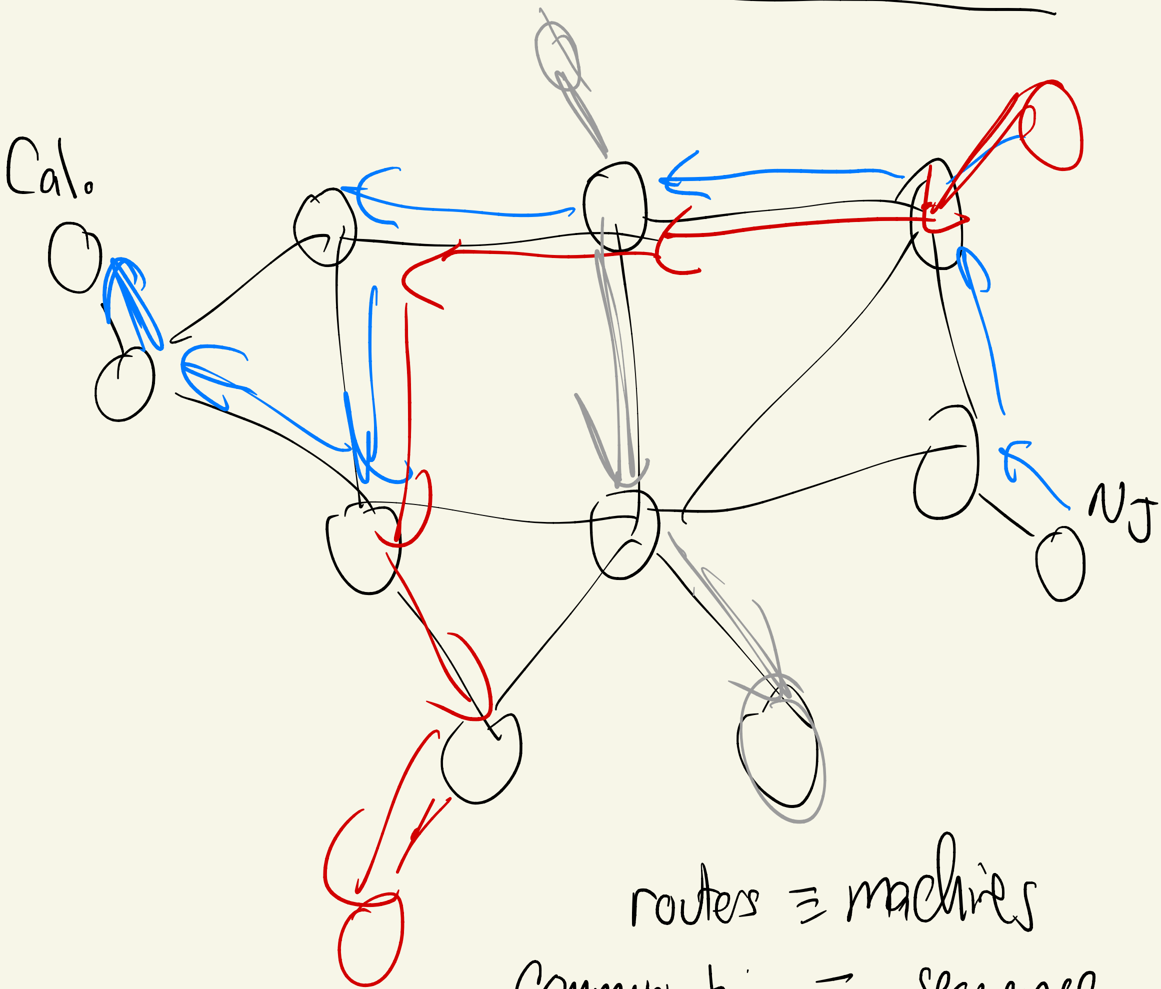
Basic Model: Multiple machines. A jobs consist of operations, each operations has a

- processing time p_{ij}
- Machine on which to run M_{ij}
- ordering information about the operations of a job

Packet Routing in Internet



routers ≡ machines

job ≡ communication ≡ sequence of machines
(e.g. email, zoomcall)

Flow Shop

$\rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow M_4 \rightarrow$

paint $\xrightarrow{>2hrs}$ stencil

heat $\xrightarrow{<5min}$ metalwork

Variants of Shop Scheduling

Basic Types

- **Job shop. Each job consist of operations in a linear order**
- **Flow shop. Job shop, but the linear order is the same for each job. (assembly line)**
- **Open shop. Each job consists of unordered operations.**

job is not a machine

Other Constraints

- **time between operations**
  - **minimum time (e.g. cooling)**
  - **maximum time (e.g. hot potato)**
- **setup at machines (e.g. paint color)**
- **limited storage between machines**

machine is not frac, a job

painting

white $\xrightarrow{0}$ white

white $\xrightarrow{10}$ black

black $\xrightarrow{30}$ white

Graduating

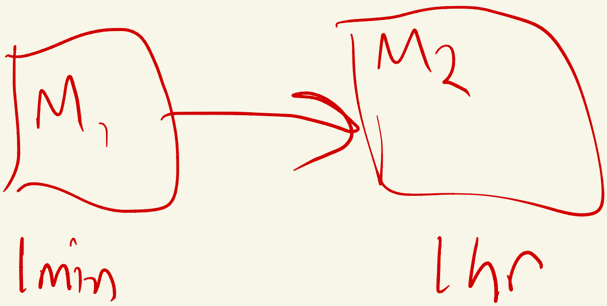
(idea of unordered tasks)

Complete Major Req

Pass Phys Ed.

Pay tuition

Get enough non-tech credits



After 1hr
1min

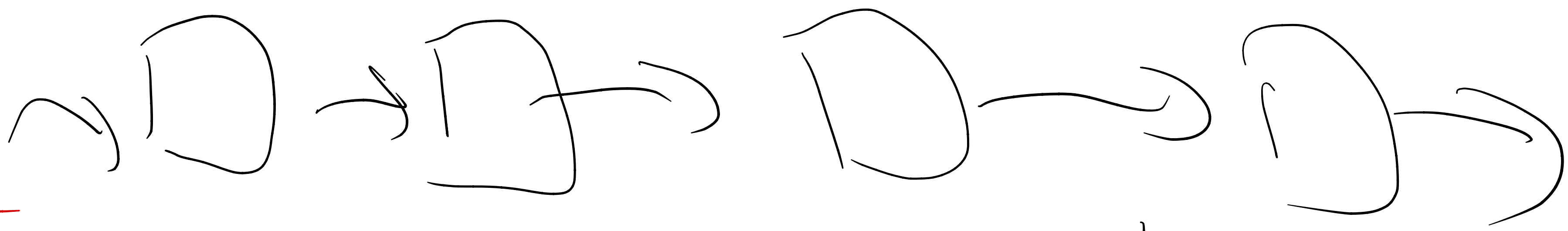
1 car proc. of M_1

58 cars sittin between M_1 & M_2

1 car processed on M_2

1 car finished

slow down the rate while jobs arrive at M_1



$J_1 \to 3 \to 6 \to$

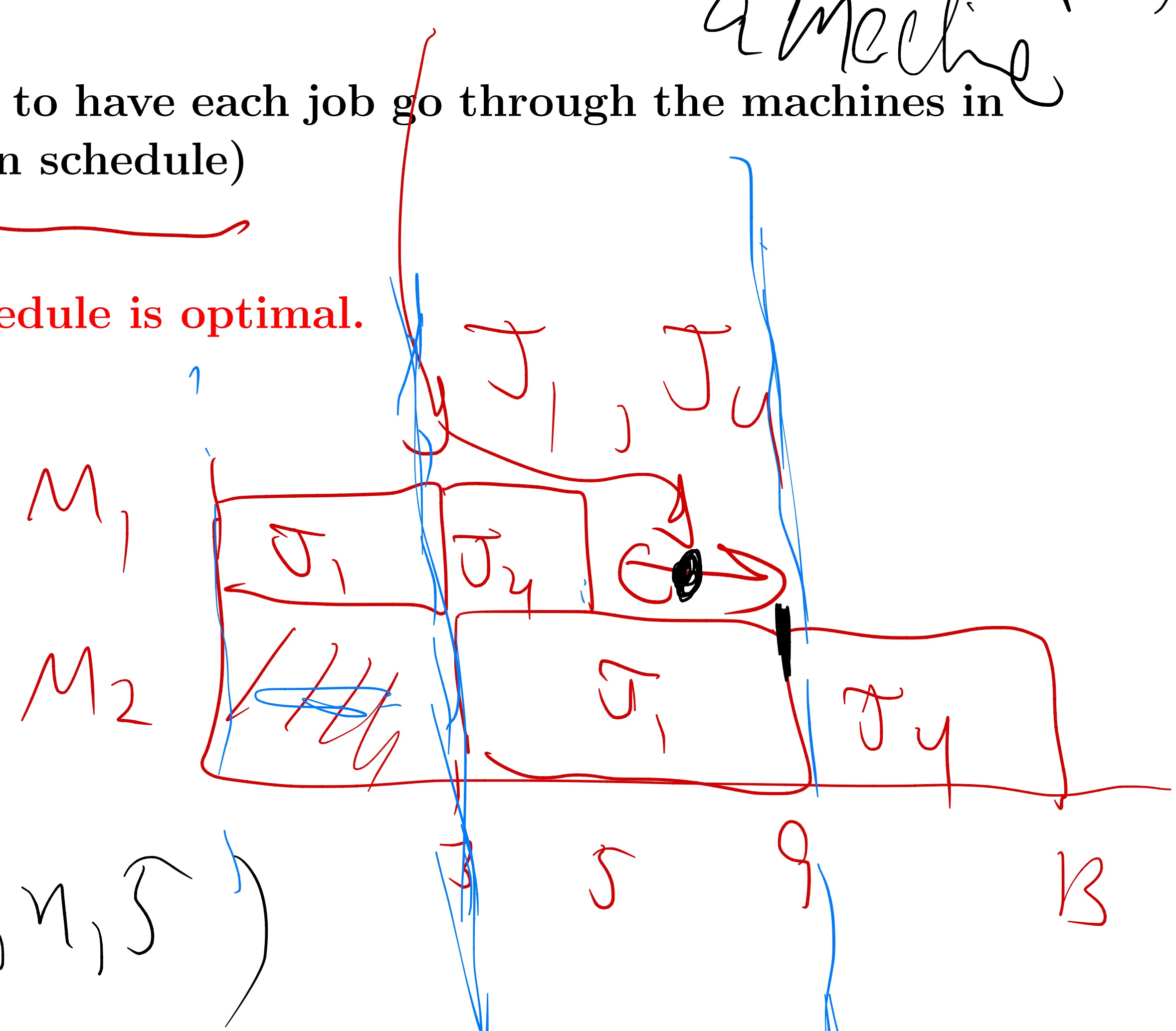
idle = machine waiting for a job

waiting time = job waiting for a machine

$F||C_{\max}$

First Question: **Is it optimal to have each job go through the machines in the same order? (permutation schedule)**

2 machines. Permutation schedule is optimal.

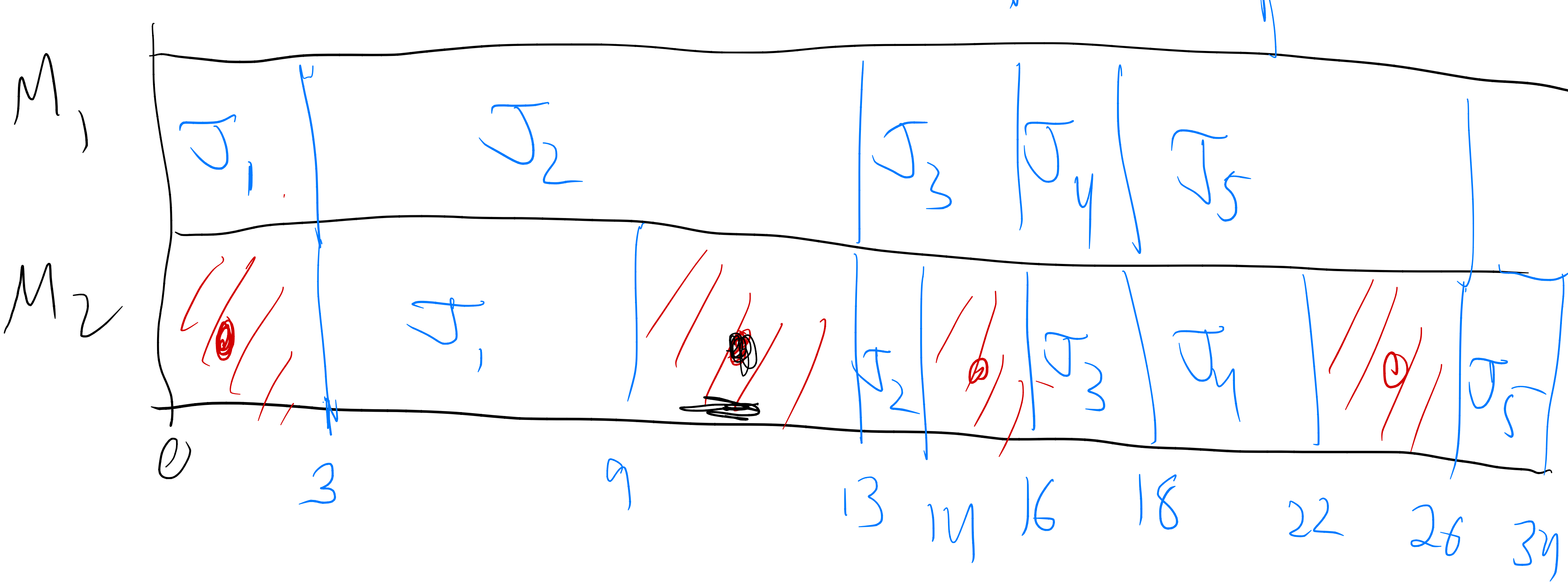


Example

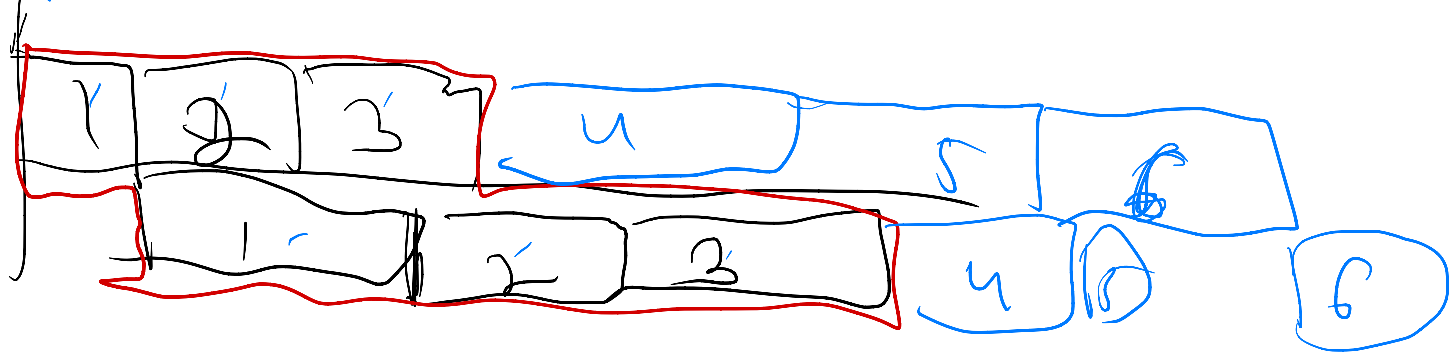
j	p_{1j}	p_{2j}
1	3	6
2	10	1
3	3	2
4	2	4
5	8	8

What is the right algorithm?

Schedule in (1, 2, 3, 4, 5)



different ex.



SPT(I)- LPT(II)

Example

j	p_{1j}	p_{2j}
1	**3**	**6**
2	**10**	**1**
3	**3**	**2**
4	**2**	**4**
5	**8**	**8**

Algorithm:

- **Partition into two sets:**
  - **Set I has** $p_{1j} \leq p_{2j}$ **(1,4,5)**
  - **Set II has** $p_{1j} > p_{2j}$ **(2,3)**
- **Run Set I in SPT order by** p_{1j}
- **Run Set II in LPT order by** p_{2j}

good to put early

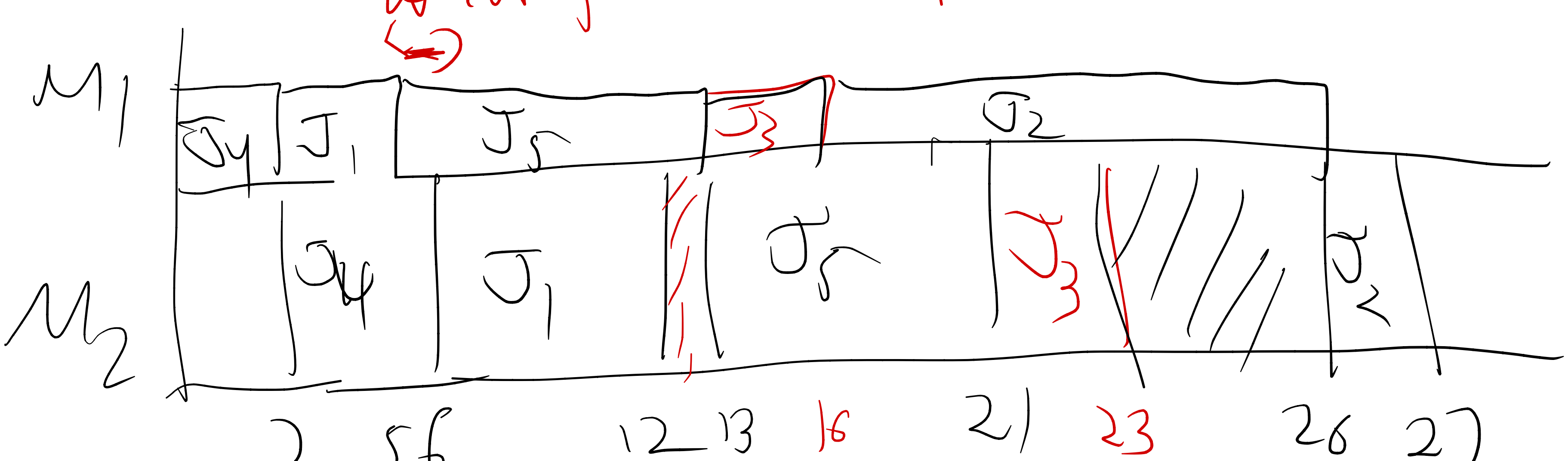
good to put late

For this problem: 4,1,5,3,2

Can use interchange arguments to show that this is optimal

- **Set I before Set II**
- **Set I in SPT order**
- **Set II in LPT order.**

waiting time for J



More general flow shop

- 3 machines. There is an optimal permutations schedule.
- 4 machines. Optimal schedule may not be a permutation schedule.

permutation schedules

$= n!$

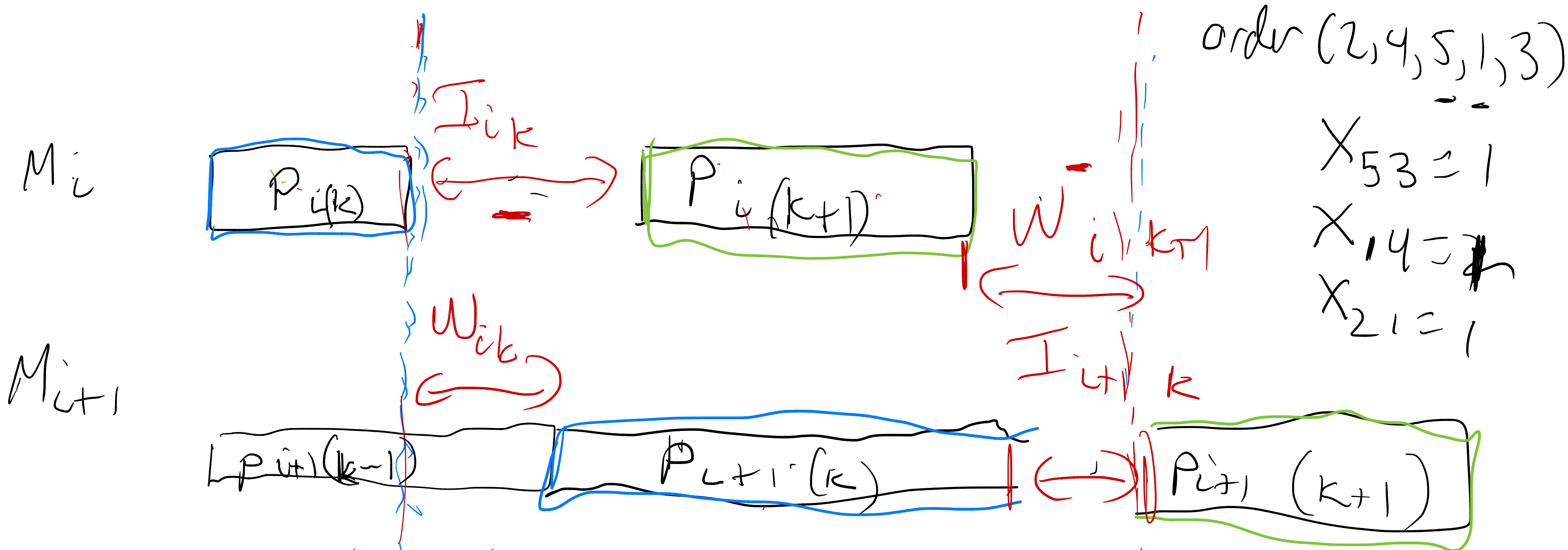
$n = 10$
$m = 10$

\# schedules =

$(n!)^m$

$n! = 10!$
$\approx 10^6$

$(n!)^m = 10^{16}$



$F|\text{perm}|C_{\max}$ as a mixed integer program

Decision variables: $x_{jk} = 1$ **if job** j **is** k **th in sequence**

processing time of the 3rd job to run on M_i

Extra Variables:

- I_{ik} **: idle time on machine** i **between jobs in positions** k **and** $k+1$ **.**
- W_{ik} **: waiting time of job in position** k **between machines** i **and** $i+1$ **.**

Ideas

- **Makespan is sum of**
  - **Processing time of first job on all machines**
  - **processing time of all jobs on machine** m
  - **Idle time on machine** m
- **Matching constraints to ensure that each job is in one position and each position has one job**
- **Relationship between idle time and waiting time constraints.**
- **Way to map variables so you can talk about** k **th job to run, rather than job indexed by** j **.**

$$P_{q(3)} = x_{13} p_{q1} + x_{23} p_{q2} + x_{33} p_{q3} + x_{43} p_{q4} + x_{53} p_{q5}$$

$= 1$

proc. time of 3rd job to run on M_q

$= p_{q5}$

MIP

n jobs

$\sim n^2$ x var's

polynomial size

Processing time of k th job to run on machine i :

$$p_{i(k)} = \sum_{j=1}^{n} x_{jk} p_{ij}$$

Objective

job 1 — machine m — machine m idle time

$$\sum_{i=1}^{m-1} p_{i(1)} + \sum_{j=1}^{n} p_{mj} + \sum_{j=1}^{n-1} I_m j$$

Matching Constraints

$$\sum_{j=1}^{n} x_{jk} = 1 \quad k = 1 \ldots n$$

— each position has a job

$$\sum_{k=1}^{n} x_{jk} = 1 \quad j = 1 \ldots n$$

each job has a position

Constraints relating idle and waiting time

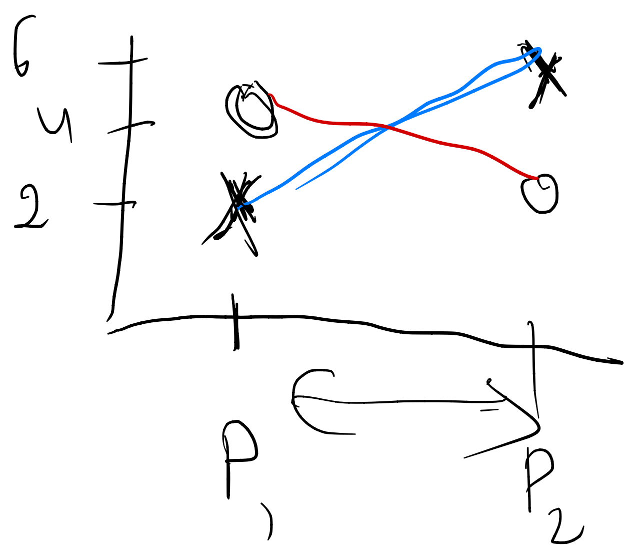
$$I_{ik} + p_{i(k+1)} + W_{i,k+1} = W_{ik} + p_{i+1(k)} + I_{i+1,k} \quad \forall k, i$$

$$W_{i1} = 0 \forall i, \quad I_{1k} = 0 \forall k$$

.

Other Facts

- $F3||C_{\max}$ is NP-complete.
- $F3|\mathbf{perm}|C_{\max}$ is NP-complete.
- Easy case: all operations are the same size. Then flowshop with many objectives is easy.



X = J_1 (2, 6) + slope

O = J_2 (4, 2) − slope

$P_2 - P_1 = (-1, +1) \cdot (P_1, P_2)$

Slope Heuristic

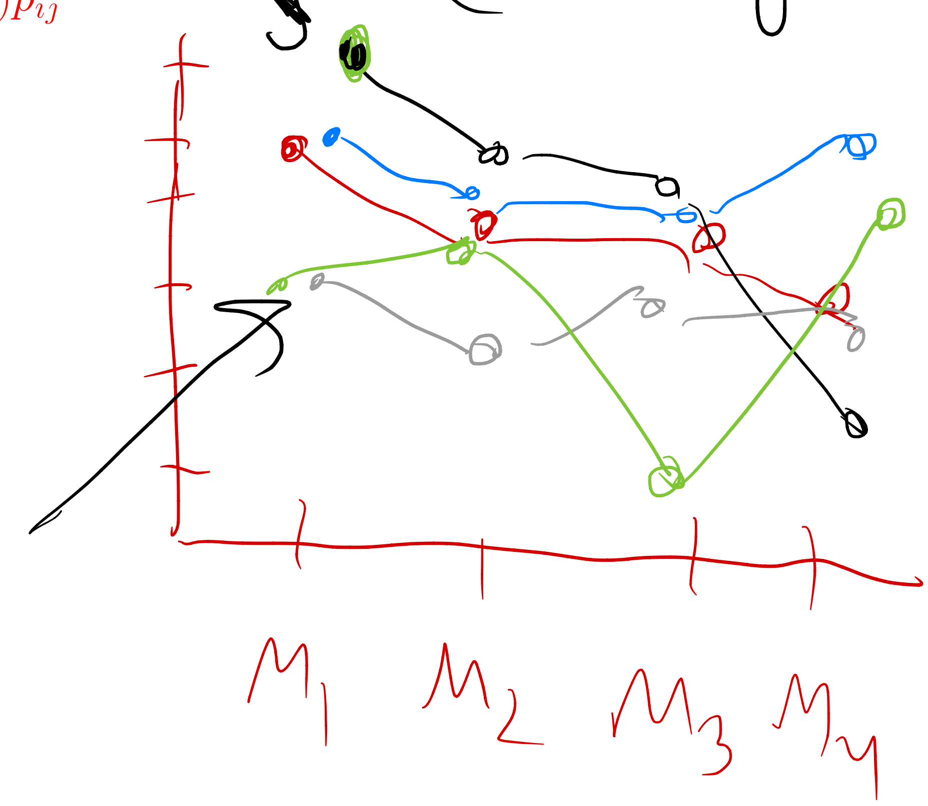
Motivation: **Think about SPT(I)-LPT(II).**

- **Early jobs should be small on** M_1 **and large on** M_2 **.**
- **Late jobs should be large on** M_1 **and small on** M_2 **.**
- **Generalize to "slope". Larger slope should go earlier.**
- **Slope** $A_j = -\Sigma_{i=1}^{m}(m-(2i-1))p_{ij}$

Alg slope

Example

	J_1	J_2	J_3	J_4	J_5
M_1	**5**	**5**	**3**	**6**	**3**
M_2	**4**	**4**	**2**	**4**	**4**
M_3	**4**	**4**	**3**	**4**	**1**
M_4	**3**	**6**	**3**	**2**	**5**

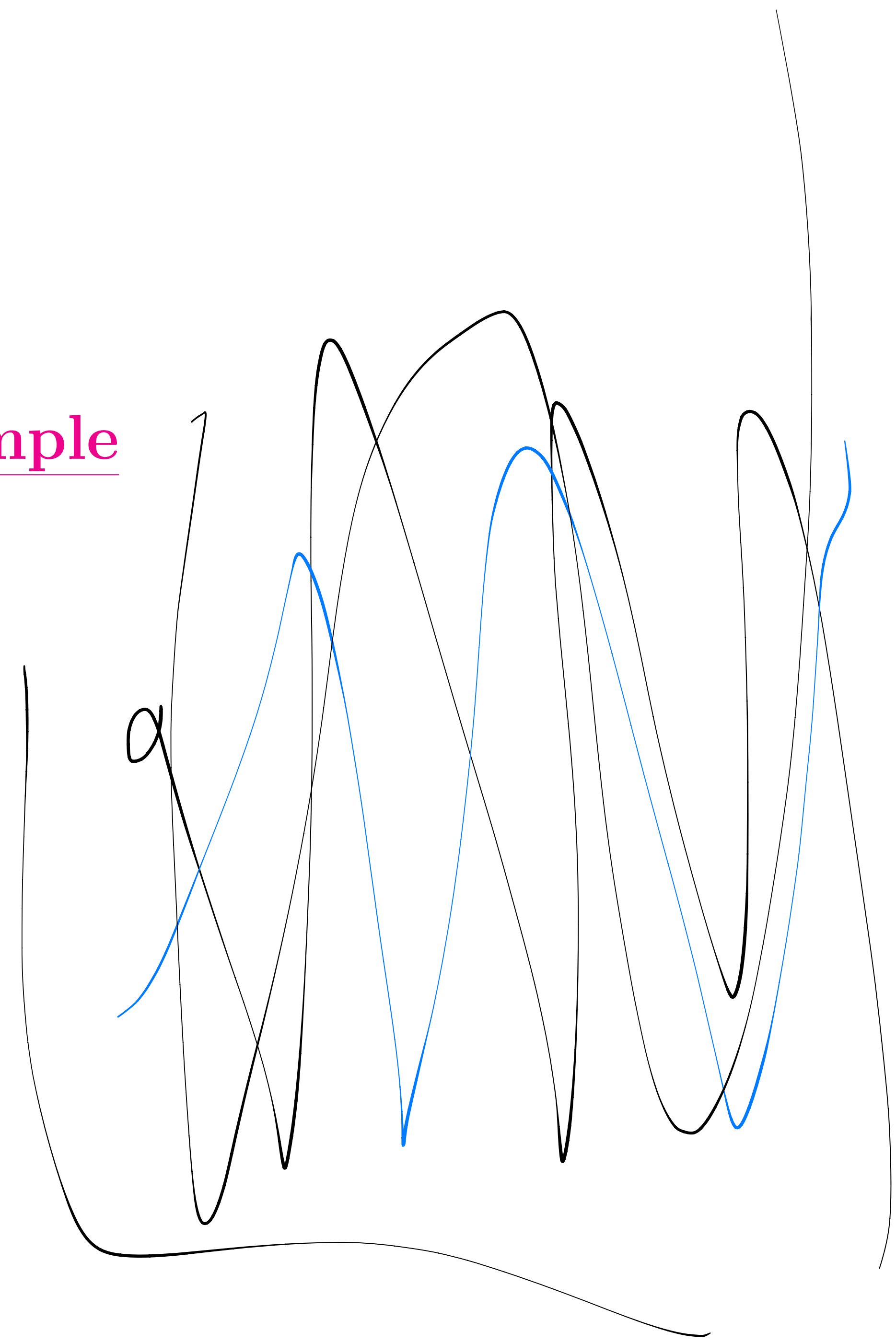


slope ≈ 0

slope of J_1

$(-3\ -1\ +1\ +3) \cdot (5\ 4\ 4\ 3)$

Example



Example

	J_1	J_2	J_3	J_4	J_5
M_1	**5**	**5**	**3**	**6**	**3**
M_2	**4**	**4**	**2**	**4**	**4**
M_3	**4**	**4**	**3**	**4**	**1**
M_4	**3**	**6**	**3**	**2**	**5**

Example:Compute Slopes

	J_1	J_2	J_3	J_4	J_5
M_1	**5**	**5**	**3**	**6**	**3**
M_2	**4**	**4**	**2**	**4**	**4**
M_3	**4**	**4**	**3**	**4**	**1**
M_4	**3**	**6**	**3**	**2**	**5**
A_j	**-6**	**3**	**1**	**-12**	**3**